

CHARACTERIZATION OF A PERIODIC SURFACE PROFILE BY POLE-ZERO
PARAMETERIZATION OF ELASTODYNAMIC PULSE REFLECTIONS*

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INTRODUCTION

This paper considers the problem of determining the height and period of a surface profile which is periodic in one direction through the analysis of reflected elastodynamic pulses. Both theory and experiment are presented.

In earlier work,^{1,2} it was noted that the periodic structure of the surface displayed a resonance at a fundamental resonant frequency, and at the subsequent harmonics. In addition, the periodic surface acts as a diffraction grating which mode converts the incident field into diffracted longitudinal and transverse spectral orders. In this paper, the "quality" of this surface resonance, and the efficiency of the diffracted order mode conversion, are quantified by modelling the reflected signal frequency spectrum as a rational polynomial over the complex frequency plane. The pole and zero loci of this rational polynomial are obtained as functions of the height and period of the surface profile, thereby providing quantitative data suitable for use in an inversion process.

EXPERIMENTAL DATA

Experimental data for scattering by several triangular periodic profiles machined in stainless steel were obtained using a pulse-echo ultrasonic technique. All profiles had a period D of 1.6mm. The following ratios of depth h to period D were considered: $h/D = 0.0, .125, .25, .375, .5, .625$. The profile geometry is depicted in Fig. 1. The longitudinal and transverse wave velocities are $c_L = 5900$ m/sec and $c_T = 3230$ m/sec. The steel samples were half-submerged in a water bath with the periodic profile exposed to air. A .75 inch, 2.25 MHz transducer was placed below the steel samples in a position for normal incidence. The output of the pulser-receiver was digitized using a digital oscilloscope, and then transferred to a main-frame computer for further processing.

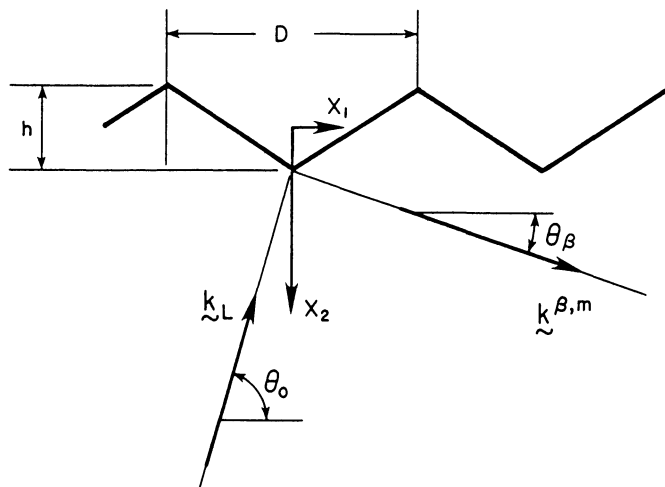


Fig. 1. Profile Configuration

The digitized time domain waveforms for three values of h/D are shown in Fig. 2. The decaying sinusoidal tail seen in the waveforms of Fig. 2 is due to the radiation of the surface resonance into the longitudinal field. In the previously cited papers^{1,2}, it is noted that this resonance occurs at approximately the frequency for which the Rayleigh wavelength equals the profile period D . Note that the amplitude and decay of the sinusoidal tail varies with h/D . The pole-zero parameterization to be presented quantifies these resonant characteristics.

THEORETICAL DATA

Theoretical frequency spectra for the reflection of a unit amplitude time harmonic longitudinal plane wave under normal incidence were obtained using a numerical solution to the boundary integral problem.³ The amplitude and phase of the reflected longitudinal field emerging normal to the surface are plotted in Fig. 3 for the values of h/D found in Fig. 2. The phase spectra display a rapid 360 degree phase shift around 1.9 MHz, indicating a surface resonance. The minima in the amplitude spectra are due to mode conversion into the $m = \pm 1$ transverse spectral orders. For example, the deep minimum in the $h/D = .375$ spectrum at 2.5 MHz implies the existence of large amplitude transverse waves propagating away from the surface at angles of $\theta = 36$ degrees and $\theta = 144$ degrees with respect to the x_1 -axis.

The spectra of Fig. 3 correspond in the time domain to the reflected response to an incident plane wave delta-function displacement. In principle, experimental deconvolved spectra, corresponding to the theoretical spectra of Fig. 3, can be obtained by applying a Fourier transform algorithm to the experimental pulses. It was shown¹, however, that large discrepancies between the theoretical and experimental spectra can arise near the resonant frequencies as a result of the electronic time gating applied to the experimental signals. The effects of time gating diminish the usefulness of the

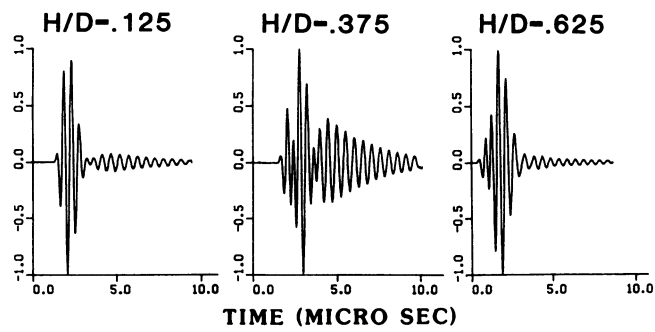


Fig. 2. Digitized time-domain waveforms for three values of h/D .

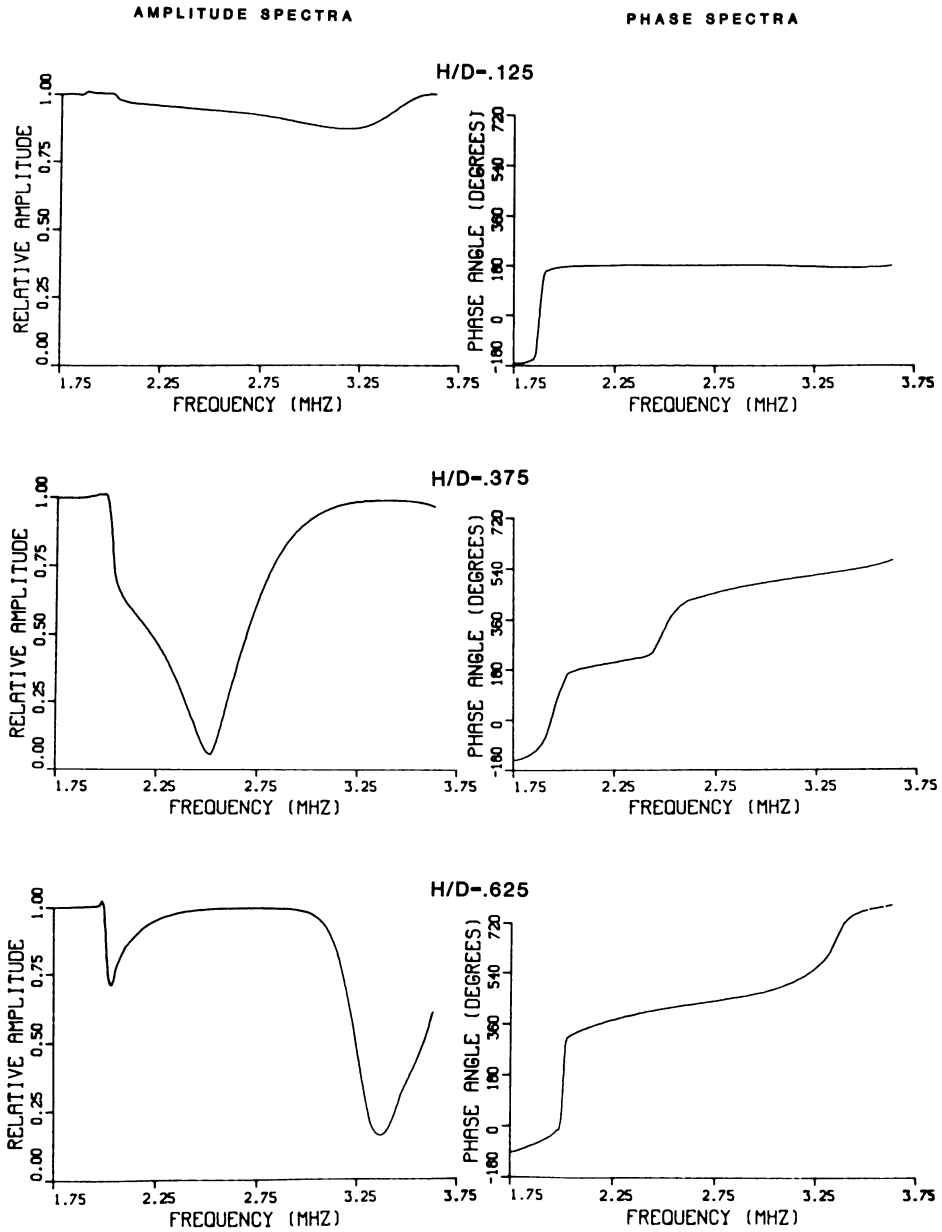


Fig. 3. Amplitude and phase spectra for three values of h/D .

experimental spectra for obtaining quantitative information. The pole-zero parameterization technique discussed in the next section operates on the experimental data in the time domain, thereby avoiding the problems associated with time gating.

It was previously noted² that the resonant decay characteristics of the reflected time domain signals display a dependence of the width of the incident beam used to insonify the reflecting surfaces. However, numerical results obtained using a theoretical model which accounted for finite beam width² revealed that, for the beam widths considered here, the errors introduced by assuming plane wave incidence are negligible.

POLE-ZERO PARAMETERIZATION

The pulse-echo experiment can be thought of as a series of cascaded electronic filters, one of which is the periodic reflecting surface. The input to this filter, $x(t)$, is defined as the pulse-echo system response to reflection from a plane surface, while the filter output, $y(t)$, is defined as the pulse-echo system response to reflection from the periodic profile. In the complex frequency (s) domain, the filter response function, $h(s)$, is defined as

$$\hat{h}(s) = \hat{y}(s)/\hat{x}(s) \quad (1)$$

where $\hat{x}(s)$, $\hat{y}(s)$ are the Laplace transforms of $x(t)$, $y(t)$, respectively. In the pole-zero parameterization technique, the response function $\hat{h}(s)$ is approximated as a rational polynomial. The poles and zeros of the approximating polynomial are parameters which depend on the geometry of the surface profile, and thus yield information useful in the inverse problem.

The pole-zero data is obtained from the experimental data by fitting a discrete recursive filter function to the digitized time domain signals via an error minimization technique.⁴ The representation of this recursive filter function in the complex frequency domain takes the form of a rational polynomial, from which the poles and zeros are extracted.

The pole-zero data is obtained from the theoretical spectra by fitting a rational polynomial directly to the complex frequency spectra using an error minimization technique.⁵

When approximating the scatterer response function as a rational polynomial, a choice must be made regarding the orders N and M of the numerator and denominator polynomials. Assume that the number of actual poles contained in the data to be analyzed is L . Then choosing $M > L$ will yield M poles; L of which correspond to actual poles in the data, and $M-L$ of which are "extraneous" poles, lying in positions which minimize the defined error criterion⁶. A similar statement applies to the zeros of the rational polynomial. It is possible to determine which of the M poles and N zeros correspond to actual poles and zeros in the data by noting the poles and zeros which repeatedly occur as N and M are increased. Another technique for recognizing actual poles and zeros in the data is to repeat the parameterization using different weighting functions in the error minimization routine for fixed N and

M, and noting the recurring poles and zeros. Both techniques were used in obtaining the pole-zero data presented here.

Two sets of complex-conjugate poles and two corresponding sets of complex-conjugate zeros were observed consistently in the parameterization of the experimental data. Corresponding pole-zero pairs were also found in the theoretical data. The coordinates in the complex frequency plane of these pole-zero pairs will be referred to as

$\alpha_j^l + i\omega_j^l$, $l = a, b$, $j = p, z$, where the superscripts "a" and "b" are used to distinguish between the two pole-zero pairs. By referring to the theoretical frequency spectra, it was noted that the pole-zero "a" had an imaginary coordinate ω_j^a , $j = p, z$, near the resonant frequency of periodic surface, while the imaginary coordinate of the pole-zero pair "b" appeared near the minima due to the mode conversion into the $m = \pm 1$ transverse spectral orders. The imaginary coordinates of these pole-zero pairs showed little variation for $N = M > 16$, while the real coordinates tended to oscillate slightly about mean values as $N = M$ increased.

INVERSION OF SCATTERING DATA USING POLE-ZERO PARAMETERIZATION

A simple means to determine the profile period from the pole-zero data, is to note that ω_j^a , $j = p, z$, lies near the surface profile resonance, hence

$$D \sim 2\pi c_{\text{Rayleigh}} / \omega_j^a, \quad j = p, z \quad (2)$$

The relation of Eq.(2) can be improved by carefully noting the shift in ω_j^a , $j = p, z$, as h/D increases, and, in the inverse process, allowing for this shift once h/D has been determined.

There are potentially several different functions which can be obtained from the pole-zero data which will display a parametric dependence on profile depth h . The function which is presented here is the difference d^l , $l = a, b$, between the real coordinates of the pole-zero pairs

$$d^l \equiv \alpha_z^l - \alpha_p^l, \quad l = a, b. \quad (3)$$

Plots showing the variation in d^l , $l = a, b$ with h/D are shown in Fig. 4. The agreement between theory and experiment is generally good, except for the $h/D = .375$ sample. It was noted that the profile of the $h/D = .375$ stainless steel sample was not truly triangular, due to improper machining. Further experiments are needed, however, to verify that this is the cause of the discrepancy.

The data of Fig. 4 provides a means for quantitative inversion of the reflected signals. Note, however, that it is implicitly assumed that the surface profile is periodic in one dimension, and has a triangular, symmetric shape, which suggests topics for future work. Future studies need to consider 1) the dependence of pole-zero loci

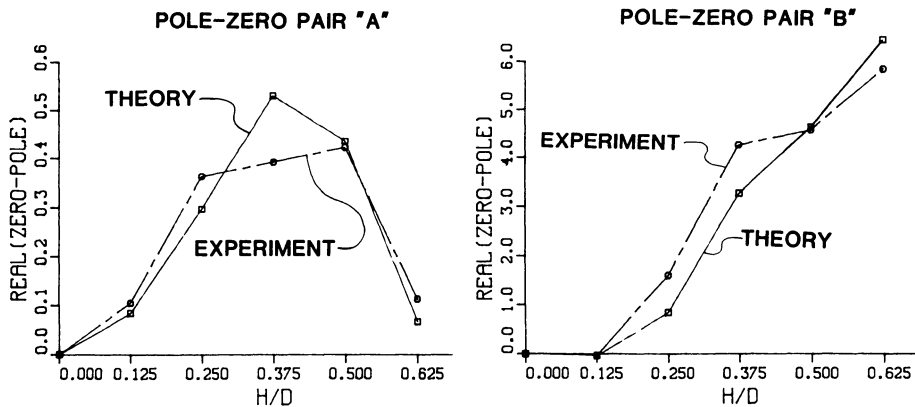


Fig. 4. Difference d as defined by Eq.(3) for two pole-zero pairs, versus h/D .

on the profile shape (e.g. sinusoidal, square, ramp, etc.), 2) the effects of deviation from true periodicity, including periodic surfaces of finite extent, and 3) the effects of periodicity in two dimensions. The results shown have encouraged the future refinement of pole-zero parameterization as a tool for scattering data inversion.

ACKNOWLEDGMENT

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